Physics 102 Summer School, Final Exam, İTÜ , 9 August 2007

- Please solve each question just below the question.
- Please show all your steps and explain your reasonings.
- No units, no points.
- Write the equation before putting in the numbers.
- Good luck!

1. **Sources of Magnetic Fields**

   a - (2pts) Write down the generalized Ampere's Law.

   $\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{in} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{E} \cdot d\vec{S}$

   b -(4pts) In an RC circuit the charge on a plate changes as $Q = \mathcal{E}C(1 - e^{-t/RC})$ where $\mathcal{E}$ is the EMF, R is the resistance and C is the capacitance. The capacitor is made of two parallel circular plates with surface are A. Find the displacement current using the following steps: (i) Write down the electric field between the capacitors. (1pts)

   $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

   (ii) Write down the flux of electric field. (1 pts)

   $\Phi_E = EA = \frac{Q}{\varepsilon_0}$

   (iii) Calculate the displacement current. (2 pts)

   $i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$

   c - (4 pts) Two current carrying wires with length l and separation d carry currents I_1 and I_2 in the same direction. (i) Calculate the magnetic force on the second wire assuming $d \ll l$ and (ii) show the directions of magnetic fields and forces on a figure.

   See the book (27-4) for figures. Use $F_2 = I_2 l B_1$ and $B_1 = \frac{\mu_0 I_1}{2\pi d}$ to find $F_2 = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l$

2. **Faraday's Law**

a -(2 pts) Write down Faraday's law.

$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S}$

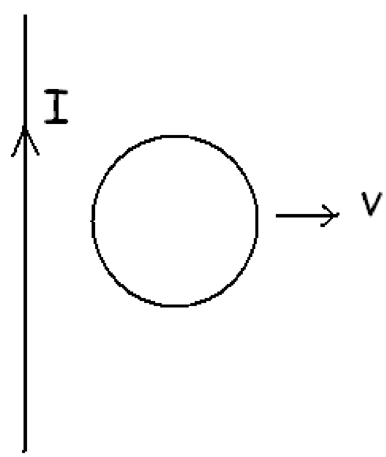
b -(5 pts) The magnitude of the magnetic field inside a solenoid is changing at a rate 4 mT/s. What is the magnitude of the induced electric field at a point inside the solenoid at a perpendicular distance of 20 mm from the axis?

Givens $r = 20mm = 0.020m$ and $dB/dt = 4 \times 10^{-3}T/s$

$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S} \Longrightarrow E2\pi r = -\frac{d}{dt}\left(B\pi r^2\right)$

$\Longrightarrow |E| = \frac{r}{2}\frac{dB}{dt} = \frac{0.02m}{2} \times 4 \times 10^{-3}T/s = 4 \times 10^{-5}$ V/m

c -(3 pts) The conducting loop is moving away from the current carrying wire. The current in the straight wire, I, is constant. Find the direction of the current induced on the loop according to Lenz's law. (In order to get points both your direction and your argument should be correct.)



As the loop is moving away the magnetic flux decreases. The induced current will flow clockwise to produce a flux trying to balance the decreasing flux.

3. **Inductance**

a -(3 pts) An infinitely long straight wire of radius R carries a current I uniformly distributed over its cross section. Determine the magnetic field B at a distance r from the axis of the wire using Ampere's law.

Uniformly distributed current $\Longrightarrow I_{in} = I\left(\frac{r}{R}\right)^2$ (note: $J = I/\pi R^2$ and $I_{in} = J\pi r^2$)thus $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in} \Longrightarrow$ $B2\pi r = \mu_0 I\left(\frac{r}{R}\right)^2 \Longrightarrow B = \frac{\mu_0 I}{2\pi R^2} r$

b -(2 pts) What is the the magnetic energy density at a distance r from the axis of the wire?

$u = \frac{B^2}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2 R^4} r^2$

c -(3 pts) What is the total magnetic energy stored in a segment of length l of the wire?

$U = \int u dV = \int_0^R u 2\pi r dr l = 2\pi l \frac{\mu_0 I^2}{8\pi^2 R^4} \int_0^R r^3 dr = \frac{\mu_0 l}{16\pi} I^2$

d -(2 pts) What is the inductance L per unit length of the wire: i.e. $\frac{L}{l}$ =?

$\frac{1}{2} L I^2 = U \Longrightarrow L = \frac{\mu_0 l}{8\pi} \Longrightarrow \frac{L}{l} = \frac{\mu_0}{8\pi}$

4. **Maxwell's Equations and Electromagnetic Waves**

Consider a sinusoidal EM wave moving in the $+y$ direction. The magnetic field oscillates with amplitude $B_0 = 2 \times 10^{-7}$ T and wavelength $\lambda = 3 \times 10^{-8}$ m in the xy plane. ($c = 3 \times 10^8$ m/s. $\mu_0 = 4\pi \times 10^{-7}$ Tm/A.)

a -(1 pts) What is the frequency, ν, of the wave?

$\nu = \frac{c}{\lambda} = \frac{3\times 10^8 m/s}{3\times 10^{-8} m} = 10^{16}$ Hz

b -(1 pts) What is the wavenumber k?

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3\times 10^{-8} m} = \frac{2\pi}{3} \times 10^8$ m^{-1}

c -(2 pts) Write the magnetic field vector, $\vec{B}$.

$\omega = 2\pi\nu$

$\vec{B} = B_0 \sin(ky - \omega t)\,\vec{i} = \left(2 \times 10^{-7} T\right) \sin\left(\frac{2\pi}{3} \times 10^8 y - 2\pi 10^{16} t\right) \vec{i}$ where y is in meters and t is in seconds.

d -(2 pts) Write the electric field vector $\vec{E}$.

$E_0 = cB_0 = 60$ V/m

$\vec{E} = E_0 \sin(ky - \omega t)\,\vec{k} = (60V/m) \sin\left(\frac{2\pi}{3} \times 10^8 y - 2\pi 10^{16} t\right) \vec{k}$ where y is in meters and t is in seconds.

e -(2 pts) Calculate the Poynting vector $\vec{S}$.

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{1}{\mu_0} E_0 B_0 \sin^2(ky - \omega t)\, \vec{k} \times \vec{i} = \frac{1}{\mu_0} E_0 B_0 \sin^2(ky - \omega t)\, \vec{j}$

f -(2 pts) Write down the average intensity, $\langle S \rangle$.

$S = \frac{1}{\mu_0} E_0 B_0 \sin^2(ky - \omega t)$

$\langle S \rangle = \frac{1}{\mu_0} E_0 B_0 \langle \sin^2(ky - \omega t) \rangle = \frac{1}{2\mu_0} E_0 B_0 = \frac{1}{8\pi \times 10^{-7} \frac{Tm}{A}} 60 \text{ V/m} \times 2 \times 10^{-7} \text{ T} = \frac{15}{\pi} \text{ W/m}^2$